INVESTIGATION 1

Motions with Tetrominoes

What Happens

Session 1: Tetrominoes Students find all possible arrangements of four squares, called *tetrominoes*. They use informal language to describe the motions they can use to overlay two shapes for comparison, to convince someone that one shape is or is not the same as another. They use a single tetromino repeatedly to make a pattern that will cover a 10-by-12 grid of squares.

Sessions 2 and 3: Slides, Flips, and Turns Students continue to explore how they can cover an area with tetrominoes. In the computer game, Tumbling Tetrominoes, they practice the three basic geometric motions—slides, flips, and turns. There are two class discussions, one about the off-computer activity, one about the computer game; you will decide the best time to hold these, depending on when students have had enough experience with the activity to reflect on the discussion questions.

Session 4: Rectangles with Different Shapes Students construct different-shaped rectangles with the same area, cutting apart 10-by-12 grids and reassembling the pieces into new rectangles. Students also work on the computer game, Tumbling Tetrominoes, with different-shaped rectangles. They discuss the "fairness" of the different shapes, touching on ideas about conservation of area and strategies for covering.

Session 5: Final Challenges Students cover their different-shaped rectangles with tetrominoes and explore whether the same tetrominoes can cover these new rectangles as covered the original 10-by-12 grid. In the computer game, students are challenged to try the Star level, in which they cannot erase a move or flip the given shapes. As an assessment, students complete a puzzle that requires them to visualize the motions they've been using.

Mathematical Emphasis

- Measuring area by covering a flat space with square units
- Finding systematically all possible geometric arrangements of a given number of squares
- Finding patterns for covering a space
- Comparing areas of rectangles with different dimensions
- Describing physical motions in precise ways as a series of slides, flips, and turns
- Comparing two shapes to decide if they are congruent or not after using geometric motions—slides, flips, and turns—to try fitting one shape exactly on top of the other

What to Plan Ahead of Time

Materials

- Interlocking cubes: 130 per pair (Sessions 1–3)
- Resealable plastic bags or envelopes: 1 per student (Session 1)
- Overhead projector, transparencies and pens (Sessions 1–2)
- Scissors, crayons, glue (Sessions 2–3, 5)
- Apple Macintosh disk, Tumbling Tetrominoes, for *Flips, Turns, and Area* (Sessions 2–5)
- Computers (Macintosh II or above) with 4 MB of internal memory (RAM) and Apple System Software 7.0 or later: 1 per pair (Sessions 2–5)
- Standard number cubes and crayons or markers (Sessions 2–3, optional)
- Transparent or masking tape (Session 4)

Other Preparation

- If you plan to provide folders in which students will save their work for the entire unit, prepare these for distribution during Session 1.
- Try to make the set of tetrominoes yourself; see the **Teacher Note,** What's an –Omino? (p. 8) for directions. (Session 1)
- Use the disk for *Flips, Turns, and Area* to install the game Tumbling Tetrominoes on each available computer (see p. 57). Try the Tumbling Tetrominoes game. (Session 2)
- If you have fewer than the recommended computers, or none at all, see the **Teacher Note,** Managing the Computer Activities (p. 9), for alternative ways to present the unit. (Session 2)
- Prepare a 5-by-24 grid for each pair of students by cutting the 10-by-12 Rectangle (p. 75) in half lengthwise and taping the short ends together. (Session 5)
- Duplicate student sheets and teaching resources (located at the end of this unit) in the following quantities. If you have Student Activity Booklets, copy only the items marked with an asterisk, including any transparencies needed.

For all sessions

10-by-12 Rectangle (p. 75): at least 11 per student, and 1 transparency.* **Note:** This grid is used throughout the investigation for many purposes. The quantity needed will vary with your classroom and the choices you make; plan to make more copies as needed.

For Session 1

Student Sheet 1, Making Tetrominoes (p. 61): 1 per student (homework)

Family letter* (p. 60): 1 per student. Remember to sign it before copying.

For Sessions 2–3

Student Sheet 2, How to Play Tumbling Tetrominoes (p. 62): 1 per pair, or 1 per computer

Student Sheet 3, Game Records (p. 64): 1 per pair

Student Sheet 4, The Perfect Cover-Up (p. 65): 1 per student (homework)

Student Sheet 5, How Many Squares? (p. 66): 1 per student (homework)

For Session 5

Student Sheet 6, Puzzle Pieces (p. 67): 1 per student

Session 1

Tetrominoes

Materials

- Interlocking cubes (130 per pair)
- 10-by-12 Rectangle (1 per pair, 2 per student, homework)
- Transparency of 10-by-12 Rectangle
- Student Sheet 1 (1 per student, homework)
- Resealable plastic bags or envelopes (1 per student, homework)
- Family letter (1 per student)
- Overhead projector and pens

What Happens

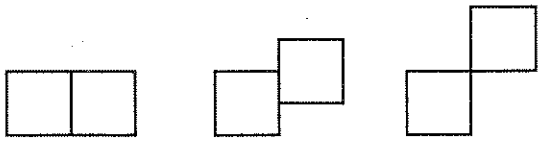
Students find all possible arrangements of four squares, called *tetrominoes*. They use informal language to describe the motions they can use to overlay two shapes for comparison, to convince someone that one shape is or is not the same as another. They use a single tetromino repeatedly to make a pattern that will cover a 10-by-12 grid of squares. Their work focuses on:

- determining when two shapes are congruent
- describing how to move a shape in order to demonstrate its congruence to another shape
- finding a pattern to cover a rectangular space

Activity

Creating Different Shapes with Four Squares

Give each student or small groups of students interlocking cubes. Challenge students to generate all possible arrangements of four squares with full sides touching. If you are substituting square tiles for cubes, you may want to demonstrate on the overhead projector what "full sides touching" means. If you are using cubes, add the rule that they must lie flat—no stacking.



OK NOT OK NOT OK

While students are working on this task, observe them and ask:

Could we give each shape a name? What do they look like to you?

Have the students invent names for the tetrominoes—such as "the L," "the T," and "the square"—and agree on what the whole class will call each one throughout the unit.

When student work has slowed down, start a whole-class discussion:

The shapes you have been making are all called *tetrominoes.* There are only a certain number of different tetromino shapes. Do you have them all? How do you know? Are some of these tetrominoes the same? Which ones? How could we prove it?

As students volunteer their ideas, encourage them to work at convincing others. They can use the overhead projector to demonstrate which shapes are the same, and how they know.

At this point, accept students' informal ideas and language as they discuss the motions they use on a tetromino to demonstrate that it is just like another; at the same time, you can use the terms *slide, flip,* and *turn* to describe their motions, as the students show what they are doing on the overhead projector.

You might also want to introduce *congruent* as the mathematical term for the idea "having exactly the same size and shape." That is, if one shape can be slid, flipped over, or turned to fit exactly on another shape, those two shapes are *congruent.* However, the emphasis in this session should not be on terminology, but on becoming familiar with the tetrominoes and ways of moving them.

Activity

Covering a Rectangle with One Shape

Hand out the 10-by-12 Rectangle, one copy to every pair of students, as you introduce the activity:

Pick your favorite tetromino, make a bunch of that shape with your cubes, and fit them together in a design that completely covers this rectangle. Try to fit as many of your particular tetromino shape as possible inside the rectangular frame, without any cubes hanging over or sticking out the sides of the rectangle. Also, remember to keep them flat—no cubes sticking up in a second layer.

As students work, ask them to think about these questions:

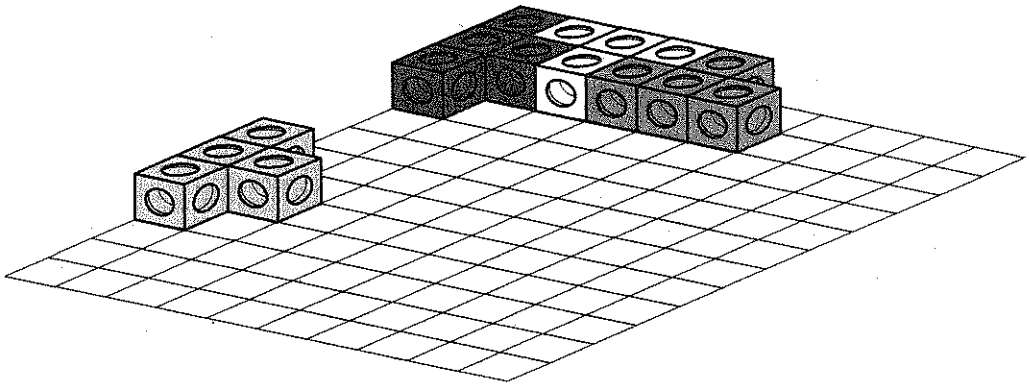
How many squares are there altogether on the grid?

How many squares did you cover with your tetromino design?

How did you figure out how many you covered?

See the **Dialogue Box,** How Many Squares? (p. 9), for a sample student discussion of these matters.

Note: Students will continue this same activity, with further discussion, in Session 2. You may want to collect the 10-by-12 Rectangles for later use.



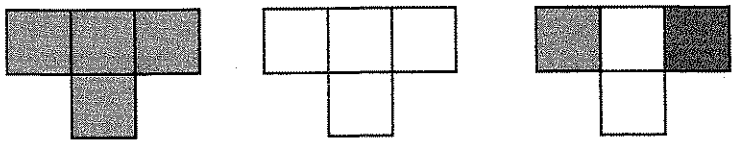
Session 1 Follow-Up

Homework

Making Tetrominoes Send home the family letter or *Investigations* at Home. Each student will also need Student Sheet 1, Making Tetrominoes, 2 copies of the 10-by-12 Rectangle, and a resealable plastic bag or envelope.

For homework, pick your favorite tetromino and make as many copies as you can from this grid paper. Color one tetromino at a time, then cut it out. Color and cut out as many as you can. You can throw away any leftover squares.

Make all four squares in each tetromino the same color. But if you want, each whole tetromino can be a different color. [*Demonstrate on the overhead with colored pens and the 10-by-12 Rectangle transparency.*]



THIS NOT THIS

Count how many you made, and store them in your bag [or envelope]. Write down how many you have, and keep this number with your tetrominoes.

❖ **Tip for the Linguistically Diverse Classroom** As you give the instructions, model what is expected. Show a copy of Student Sheet 1 with one of the tetrominoes drawn many times. Demonstrate coloring one tetromino and cutting it out. To demonstrate that students can switch colors, but not within a single shape, pick another crayon and color half of a second tetromino. Pause and pick a third color, as if you are going to use it on the other half, but then stop, shaking your head. Put the third color back and hold up the same crayon you started coloring this shape with, nod happily, and finish coloring the tetromino.

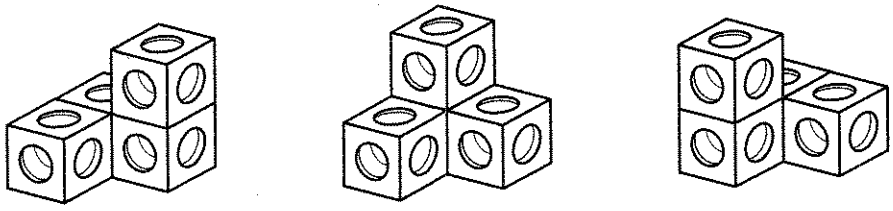
Advise students that if they don't have scissors at home, they can do just the coloring; they can complete the cutting the next day in school.

Note: Students will continue working on this activity during the next two class sessions.

Extensions

3-D Tetrominoes Suggest that students explore the set of three-dimensional tetrominoes—all the shapes that can be made with four cubes, but *without* the constraint that they must lie flat. How many of these can they find?

Solution: There are three additional tetrominoes when 3-D shapes are allowed.

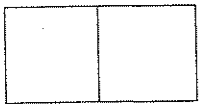


Floor Tetrominoes In one classroom with a checkerboard square pattern on the floor, students chalked tetrominoes on the floor and built large paper tetrominoes to fit over the squares, even covering a 10-by-12 rectangle with them. This extension is especially good for a class that enjoys large-motor activities.

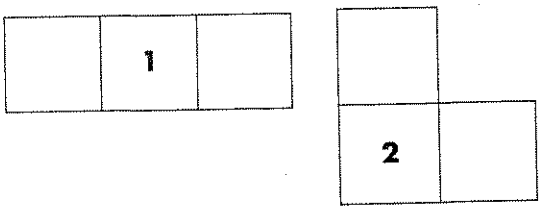
Teacher Note

What's an –Omino?

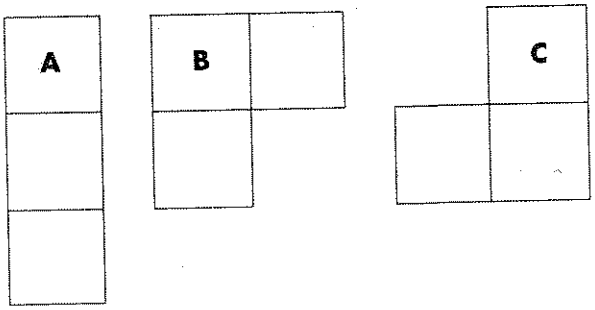
A *domino,* as you may know, is the shape formed by putting two squares together, with full edges touching. There is only one shape you can make with two squares:



Adding a third square makes a *triomino*. A triomino has two possible arrangements:



Triominoes A, B, and C may look like other possible arrangements, but they are actually the same shape as Triominoes 1 or 2.



That is, we can move these shapes through space, with a *flip* (reflection) or a *turn* (rotation), and then a slide, to exactly match Triomino 1 or Triomino 2.

To see firsthand how this works, make Triomino B out of interlocking cubes or cut it from grid paper. Watch how you move it to make it look exactly like Triomino 2. Is there more than one series of motions that will work? Triomino 2 and Triomino B are congruent—the same size and shape. What about the other triominoes?

You should have discovered that Triomino B becomes Triomino 2 with a flip and a slide. Triomino A becomes Triomino 1 with a turn and a slide. And Triomino C becomes Triomino 2 with a 90-degree turn and a slide.

Now try making the set of *tetrominoes*—all the possible arrangements of *four* squares with full sides touching. Remember, if you can move a tetromino through a series of slides, flips, and turns so that it exactly matches another tetromino, it is not a different arrangement. When you think you have all the possible tetrominoes, compare yours with the pictures on Student Sheet 1, Making Tetrominoes (p. 61). Can you convince yourself that there cannot possibly be any other tetrominoes?

DIALOGUE BOX

How Many Squares?

After students have spent some time trying to cover the 10-by-12 Rectangle with cube tetrominoes, they invent a variety of strategies for determining the total number of squares in the rectangle and the number of squares they have covered, as demonstrated in the following discussion.

How did you figure out the number of squares there are in the rectangle?

Sean: There's 118 squares. I counted every one.

Midori: No, it's 120. I took two towers of 5 on the bottom, and that was 10. There were 10 tens; 100. Then I counted by twos for the leftover columns and got 20. Then 100 and 20 is 120.

Tamara: I just counted by 10's: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120.

Samir: I did it quicker by multiplying. See, there's 10 in every row. So, you just see that there's 12 of those, $10 \times 12 = 120$.

How did you figure out how many of those squares you covered?

Sean: I just counted all the squares. I covered 116.

Tamara: I knew there were 120 in all. So, I did it mathematically. I left 8 holes, so 120 take away 8 was . . . 112.

Samir: There's another way. I counted the number of tetrominoes I used to cover the rectangle. I used the "I" tetromino and it took 30 of them to cover the whole rectangle. Each tetromino is made of 4 little squares, so 30×4 is 120. See? I covered the whole rectangle!

Managing the Computer Activities

Teacher Note

The availability of computers to your students will determine how you organize the work in Sessions 2 through 5. Regardless of the number of computers available, have students work in pairs. This not only maximizes computer resources, but also encourages students to consult, monitor, and teach one another.

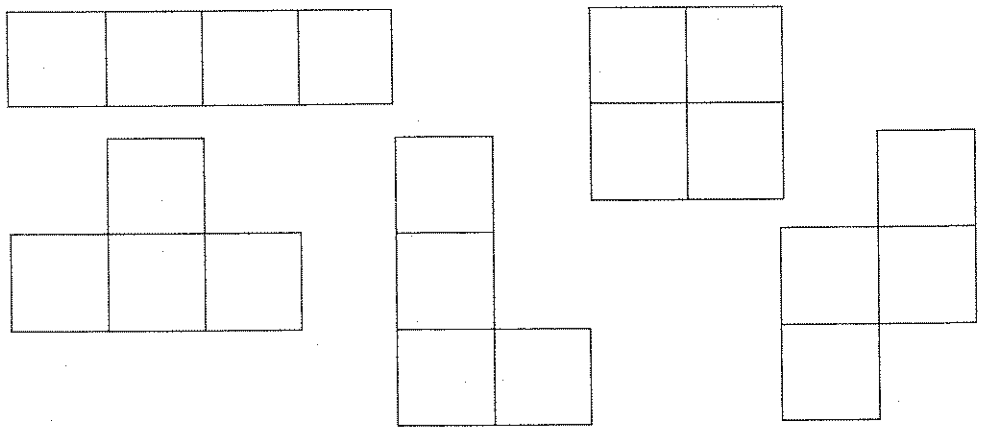
Computer Lab If you have a computer laboratory with one Apple Macintosh computer for each pair of students, you can conduct the activities in sequence. That is, you would begin Session 2 in class with the activity The Perfect Cover-Up, then move to the computer laboratory for Tumbling Tetrominoes.

Fewer Computers If you have fewer computers—from one to six in your classroom—you might want to use a choices or stations approach. To start Session 2 you would introduce both the off-computer activity, The Perfect Cover-Up, and the computer game, Tumbling Tetrominoes, to the whole class. Then, throughout Sessions 2 through 5, pairs of students would cycle through the computer game while the others work on the off-computer activities. Cycling pairs through the computer game throughout the school day, instead of just during your math hour, may be necessary to give everyone a chance at the computer within a reasonable number of days. Each pair should spend at least 20 minutes at the computer.

No Computers If you do not have a computer, you can use the activity Tumbling Tetrominoes on Paper (p. 14) as an alternative to the computer version.

Computer Help It helps to have reminders about particular keys and commands posted near the equipment. Establish rules for getting help from peers; one rule is to ask two other students before asking the teacher. Or, you might designate certain students as class computer consultants.

Making Tetrominoes



Choose your favorite tetromino shape.
Use the 10-by-12 Rectangle to make many copies of that one shape. You will use your shapes to cover rectangles.

Color each tetromino. Make all the squares in one tetromino the same color. But you can use lots of colors. You could make a red tetromino, a blue tetromino, a purple tetromino, and so forth.

Cut out each tetromino. If you don't have scissors at home, you can cut out the tetrominoes in class. Store your tetrominoes in a bag or envelope.

Count them. Write the number you have, and keep it with your shapes.

You need to make at least 30. If you don't finish, you will have time to make more in class.